Working Paper

Department of Finance

Economic and Fiscal Policy Branch

Working papers are circulated in the language of preparation, to make analytic work undertaken by the staff available to a wider readership. They have received only limited evaluation and do not reflect the views of the Department of Finance. Comments on the working papers are invited and may be sent to the author(s).

101-06865/

Do Mechanical Filters Provide a Good Approximation of Business Cycles?

by

Alain Guay
Finance Canada, CREFE and UQAM
and
Pierre St-Amant
International Department of the Bank of Canada

Working Paper 96-2
May 1996

The authors would like to thank Paul Beaudry, Pierre Duguay, Robert Lafrance, John Murray, Alain Paquet, Jacques Raynault and Simon van Norden for useful comments and discussions. Of course, since the authors are solely responsible for the paper's content, none of the above mentioned are responsible for any of their errors.

*Pierre St-Amant: Tel: 613-782-7386, E-mail: pstamant@bank-banque-canada.ca

ABSTRACT

In this paper, the authors examine how well the Hodrick-Prescott filter (HP) and the band-pass filter (BK) recently proposed by Baxter and King (1995) extract business cycle frequencies for the level of macroeconomic time series. They argue that the conclusion depends on whether one is interested in these frequencies for the difference, the level, or an unobserved cyclical component of time series. In the case of series in difference, the authors find that the BK filter greatly amplifies business cycle frequencies and creates spurious cycles. Cogley and Nason (1995) and Harvey and Jaeger (1993) reached the same result in the case of the HP filter. In the case of series in level, the authors find that the HP and BK filters reduce the importance of business cycles frequencies and give a cyclical component having a peak in its spectrum which is absent from the spectrum of the original series. The authors then use artificial data to assess the possibility that the HP and BK filters provide a good approximation of an unobserved cyclical component of macroeconomic series. They find that, when series are dominated by their permanent component, the HP and BK filters provide a distorted cyclical component. These filters perform adequately when the spectrum of the unfiltered series has a peak at business cycle frequencies. Since the spectrums of most macroeconomic series have the typical Granger shape, the authors conclude that the conditions required to obtain a good approximation of the cyclical with the HP and BK filters are rarely met in practice.

RÉSUMÉ

Cette étude vise à déterminer dans quelle mesure le filtre Hodrick-Prescott (HP) et un filtre récemment mis de l'avant par Baxter et King (1995) permettent de bien extraire les fréquences du cycle d'affaire des séries temporelles macroéconomiques. Les auteurs expliquent que la conclusion dépend de ce que l'utilisateur de ces filtres s'intéresse aux fréquences du cycle économique pour les séries en différence, en niveau, ou pour une composante cyclique non-observée. Dans le cas des séries en différence, les auteurs trouvent que le filtre de Baxter et King (BK) tend à amplifier fortement les fréquences du cycle d'affaire et à induire une dynamique fausse. Cogley et Nason (1995) et Harvey et Jaeger (1993) arrivent aux mêmes conclusions pour le filtre HP. Dans le cas des séries en niveau, les auteurs trouvent que les filtres HP et BK réduisent l'importance des fréquences du cycle d'affaire et donnent une composante cyclique dont le spectre a un maximum qui est absent du spectre de la série originale. Les auteurs font ensuite des simulations afin d'évaluer la possibilité que les filtres HP et BK puissent fournir une bonne approximation d'une composante cyclique non-observée des séries macroéconomiques. Ils trouvent que les filtres HP et BK se comportent mal dans le cas de séries dominées par leur composante permanente. Ces filtres se comportent mieux dans le cas de séries dont le spectre a un maximum aux fréquences du cycle d'affaire. Étant donné que le spectre de la plupart des séries macroéconomiques a la forme typique de Granger, les auteurs concluent que les conditions requises pour que les filtres HP et BK se comportent bien sont rarement réunies en pratique.

1. INTRODUCTION

Establishing the "stylized facts" associated with the business cycle component of macroeconomic series is a critical element in business cycle research. Since the influential paper of Nelson and Plosser (1982) which concluded that most macroeconomic series are characterized by a stochastic rather than linear deterministic trend, methods for stochastic detrending have been developed. In particular, there has been an increasing use of mechanical filters to identify permanent and cyclical components of time series, the most popular being the Hodrick-Prescott (1980) filter (HP). More recently, Baxter and King (1995) have proposed a band-pass filter (BK) whose purpose is to isolate certain frequencies in the data.[1]

The use of these filters to identify permanent and cyclical components has been criticized, however. For instance, Harvey and Jaeger (1993) and Cogley and Nason (1995) show that spurious cyclicality can be induced when the HP filter is applied to a random walk process. Osborn (1995) reports similar results for a simple moving average detrending filter. Despite these critiques, a number of researchers still use mechanical filters to retrieve permanent and cyclical components of macroeconomic time series. One possible reason is that critics based their analysis on specific data generating processes, e.g. random walks, and compared filtered series to the series in difference. However, one might ask how well filters do when applied to other processes in extracting certain frequencies for the series in level or in approximating some unobserved cyclical component.

The remainder of the paper is organized as follows. In Section 2, we show that the critiques formulated by Harvey and Jaeger, Cogley and Nason and Osborn also apply to the BK filter if one is comparing filtered series to series in differences. In Section 3, we examine how well the HP and BK filters extract the range of periodicities they were designed for in the case of series in level. We retain the definition of business cycles proposed by Burns and Mitchell (1946) and adopted by Baxter and King, that is business cycles are cyclical components of no less than six and no more than thirty-two quarters. We compare the spectrum of unfiltered series at these frequencies with the spectrum of their filtered counterparts.

Our main conclusion is the following. The HP and BK filters do well in terms of extracting business cycle frequencies of time series whose spectrum

1. This filter has already been used in several empirical studies. See Baxter (1994), King et al. (1995), and Cecchetti and Kashyap (1995). Other band-pass filters have also been proposed. See, for example, Hasler et al. (1994).

has a peak at those frequencies. Unfortunately, the peak of the spectral density of most macroeconomic series in levels is at lower frequencies. Indeed, it is well known that macroeconomic aggregates have the typical spectral shape identified by Granger (1966). Such series have most of their power at low frequencies and their spectrum decreases sharply and monotonically at higher frequencies. For such series, the HP and BK filters perform poorly in terms of extracting business cycle frequencies. The intuition behind this result is simple. The problem is that a large fraction of the power of typical macroeconomic time series at business cycle frequencies is concentrated in the band where the squared gain of HP and BK filters differ from that of an ideal filter. Also, the shape of the squared gain of those filters, when applied to typical macroeconomic time series, induces a peak in the spectrum of the cyclical component which is absent from the original series. Consequences of applying the HP and BK filters is to induce spurious dynamic properties and the extraction of a cyclical component which does not capture an important fraction of the variance contained in business cycle frequencies of the original series.

However, macroeconomic time series are often seen as comprising permanent and cyclical components. While the HP and BK filters do not provide a good approximation of business cycle frequencies for series in level, they could still provide a good approximation of an hypothetical unobserved cyclical component at business cycle frequencies if this component was characterized by a peak in its spectrum at these frequencies. We explore this possibility with artificial data in Section 4. We conclude that, when the series are dominated by the permanent component, the HP and BK filters provide a distorted cyclical component. These filters perform adequately, however, when the spectrum of the original series (including the permanent and cyclical components) has a peak at business cycle frequencies. Since most macroeconomic series have the typical Granger shape, the conditions required to obtain a good approximation of the cyclical component with the HP and BK filters are rarely met in practice. As a result, we argue that the HP and BK filters perform poorly in terms of identifying the cyclical component of most macroeconomic series.

In Section 5, we compare the cyclical component resulting from the application of the HP and BK filters to the ones obtained with detrending methods proposed by Watson (1986) and Cochrane (1994).

2. THE HP AND BK FILTERS

2.1 THE HP FILTER

The HP filter decomposes a time series y_t into an additive cyclical component (y_t^c) and a growth component (y_t^g),

$$y_t = y_t^g + y_t^c$$

The application of the HP filter consists of minimizing the variance of the cyclical component y_t^c subject to a penalty for the variation in the second difference of the growth component y_t^g,

$$\{y_t^g\}_{t=0}^{T+1} = argmin \sum_{t=1}^{T} \left[\left(y_t - y_t^g\right)^2 + \lambda \left[(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g) \right]^2\right]$$

where the parameter λ penalizes the variability in the growth component. The larger the value of λ, the smoother is the growth component. As λ approaches infinity, the growth component corresponds to a linear time trend. For quarterly data, Hodrick and Prescott propose to set $\lambda = 1600$. King and Rebelo (1993) show that the HP filter can render stationary any integrated process of up to the fourth order.

A number of authors have studied the HP filter's basic properties. As shown by Harvey and Jaeger (1993) and King and Rebelo (1993), the HP filter can be rationalized as the optimal linear filter of the trend component for the following process:[2]

$$y_t = \mu_t + \varepsilon_t$$

where ε_t is an $NID\left(0, \sigma_\varepsilon^2\right)$ irregular component and the trend component, μ_t, is defined by

$$\mu_t = \mu_{t-1} + \beta_{t-1}$$

2. That is the filter which minimizes the mean square error $MSE = (1/T) \sum_{t=1}^{T} (\hat{y}_t^c - y_t^c)^2$, where y^c_t is the true cyclical component and $\hat{y}_t^c$ is its estimate.

$$\beta_t = \beta_{t-1} + \zeta_t,$$

with $\zeta_t \sim NID(0,\sigma^2)$. β_t is the slope of the process and ζ_t is independent of the irregular component. Note that this trend component is integrated of order two, i.e., stationary in second differences.

We believe that the use of the HP filter to identify the cyclical component of most macroeconomic time series cannot be justified on the basis of optimal filtering arguments since the following critical assumptions are unlikely to be satisfied.

(1) *Transitory and trend components are not correlated with each other.* This implies that the growth and cyclical components of a time series are assumed to be generated by distinct economic forces, which is often incompatible with business cycle models -- see Singleton (1988) for a discussion.

(2) *The process y_t is integrated of order 2 -- I(2).* This is often incompatible with priors on macroeconomic time series. For example, it is usually assumed that real GDP is I(1) or I(0) around a breaking trend.

(3) *The transitory component is white noise.* This is also questionable. For example, it is unlikely that the stationary component of output is strictly white noise. King and Rebelo (1993) show that this condition can be replaced by the following assumption: *an identical dynamic mechanism propagates changes in the trend component and innovations to the cyclical component.* However, the later is also very restrictive.

(4) *The user is not interested in data points near the beginning or the end of the sample.* This is simply a consequence of the fact that the HP filter, a two-sided filter, changes its nature and becomes closer to a one-sided filter as it gets near the beginning or the end of a time series. Indeed, after studying the properties of the HP filter at those extremities, Baxter and King (1995) recommend that three years of data be dropped at both ends of a time series when the HP filter is applied to quarterly or annual data.[3]

(5) *The parameter controlling the smoothness of the trend component, λ, is appropriate.* Note that λ corresponds to the ratio of the variance of the irregular component to that of the trend component. Economic theory provides little or no guidance on what this ratio should be. While attempts have been made to estimate this parameter via maximum likelihood methods -- see Harvey and Jaeger (1993) or Côté and Hostland (1993) -- indications are that it might not be possible to estimate λ with reasonable

3. This is clearly a problem for policy makers who would want to use the HP filter to estimate current potential output. This is discussed in Laxton and Tetlow (1992) and van Norden (1995).

precision.

Despite these shortcomings, Singleton (1988) shows that the HP filter can nevertheless be a good approximation of a high-pass filter when it is applied to stationary time series. In order to discuss this we need to introduce some elements of spectral analysis. A zero mean stationary process has a Cramer representation such as:

$$y_t = \int_{-\pi}^{\pi} \varepsilon(\omega)\, d\omega$$

A stationary time series y_t can thus be expressed as the integral of random periodic components, $\varepsilon(\omega)$, which are mutually orthogonal. In turn, filtered time series can be expressed as

$$y_t^f = \int_{-\pi}^{\pi} \alpha(\omega)\, \varepsilon(\omega)\, d\omega$$

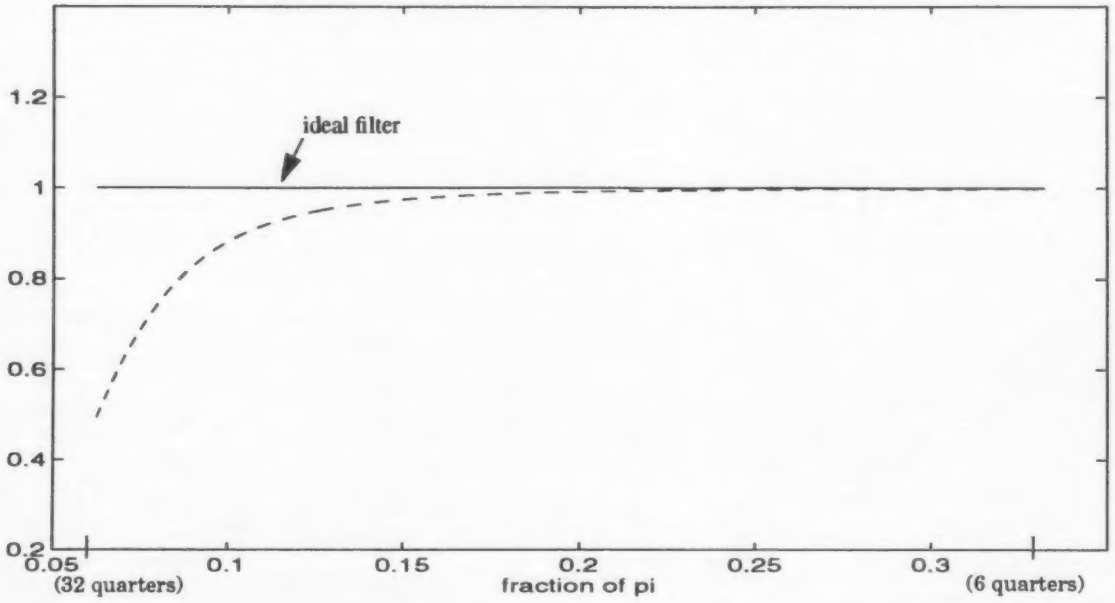
with

$$\alpha(\omega) = \sum_{h=-k}^{k} a_h e^{-i\omega h} \qquad (1)$$

where i is the imaginary number ($\sqrt{(-1)}$) and ω is frequency measured in radians, i.e., $-\pi \leq \omega \leq \pi$. Equation (1) is the frequency response (Fourier transform) of the filter. That is, $\alpha(\omega)$ indicates the extent to which y_t^f responds to y_t at frequency ω and can be seen as the weight attached to the periodic component $\varepsilon(\omega)$. In the case of symmetric filters, the Fourier transform is also called the gain of the filter.

An ideal high-pass filter would remove low frequencies or long cycle components and allow for high frequencies or short cycle components to pass through, so that $\alpha(\omega) = 0$ for $|\omega| \leq \omega^p$ where ω^p has some predetermined value, and $\alpha(\omega) = 1$ for $|\omega| > \omega^p$. Chart 1 shows the squared gain of the HP filter. We see that the squared gain is zero at zero frequency and is close to one from around frequency $\pi/10$ and at higher frequencies. Thus, the HP filter appears to be a good approximation of a high-pass filter in that it removes low frequencies and passes through higher frequencies.

Chart 1: squared gain of the HP filter



An important problem with this is that most macroeconomic time series are either integrated of highly persistent processes so that they are better characterized in small samples as non-stationary rather than stationary processes. In their study of the implications of applying the HP filter to integrated or highly persistent time series, Cogley and Nason (1995) argue that the HP filter is equivalent to a two-step linear filter which would initially first difference the data to make them stationary and then smooth the differenced data with the resulting asymmetric filter. The filter tends to amplify cycles at business cycle frequencies in the detrended data and to dampen long and short-run fluctuations. Cogley and Nason conclude that the filter can generate business cycle periodicity even if none is present in the data.[4] Harvey and Jaeger (1993) make the same point. To better understand this result, consider the following I(1) process

$$(1-L)\,y_t = \varepsilon_t \tag{2}$$

where ε_t is zero mean and stationary. King and Rebelo (1993) show that the HP cyclical filter can be rewritten as $(1-L)^4 H(L)$. We define $|HP(\omega)|^2$ as the squared

4. Classic examples of filter induced cyclicality in the context of stationary time series are Slustsky (1937) and Howrey's (1968) analysis of Kuznets' transformations. These examples are discussed in Chapter 11 of Sargent (1987).

gain corresponding to the HP cyclical filter where $HP(\omega)$ is the Fourier transform of $(1-L)^4H(L)$ at frequency ω. When the HP filter is applied to the level of the series y_t, the spectrum of the cyclical component is defined as

$$f_{y^c}(\omega) = |HP(\omega)|^2|1-\exp(-i\omega)|^{-2}f_\varepsilon(\omega)$$

where $(1-\exp(-i\omega))$ is the Fourier transform of $(1-L)$ and $f_\varepsilon(\omega)$ is the spectrum of ε_t which is well defined since ε_t is a stationary process. Obviously, $|1-\exp(-i\omega)|^{-2}$ is not defined for $\omega = 0$. The expression $|1-\exp(-i\omega)|^{-2}f_\varepsilon(\omega)$ is often called the pseudo-spectrum of y_t.

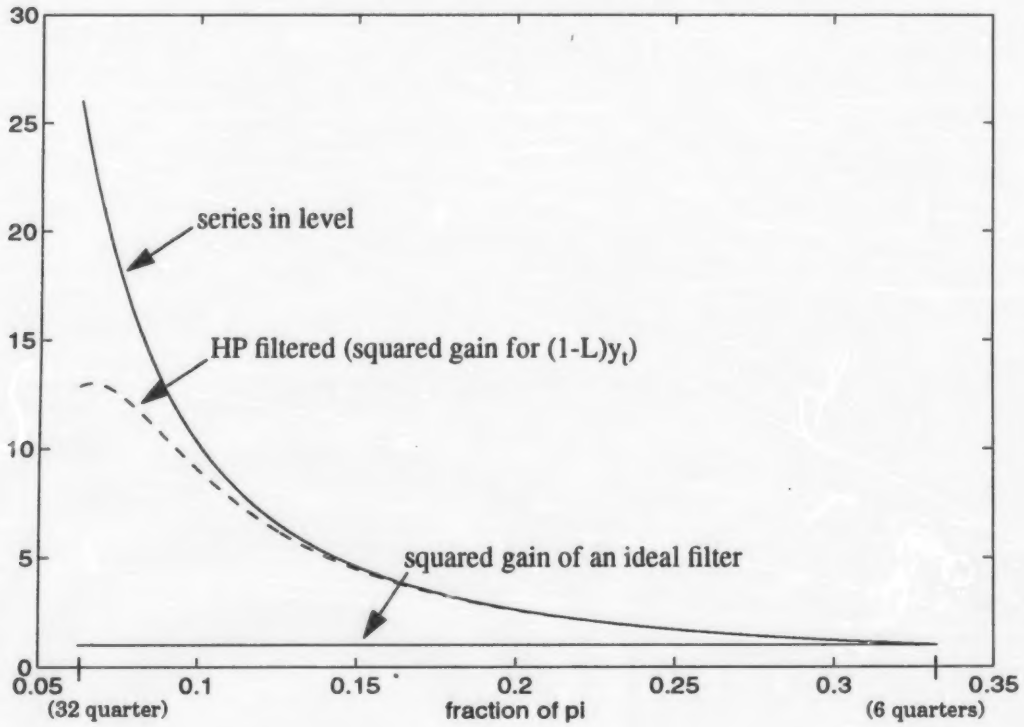
Cogley and Nason and Harvey and Jaeger calculate the squared gain of the HP cyclical component for $(1-L)y_t$, which corresponds to the squared gain of $(1-L)^3H(L)$ since $(1-L)^4H(L)y_t = (1-L)^3H(L)(1-L)y_t$. The squared gain corresponding to the filter applied to $(1-L)y_t$ is then equal to $|HP(\omega)|^2|1-\exp(-i\omega)|^{-2}$. They show that applying the HP filter to a random walk produces detrended series having the characteristics of a business cycle for quarterly data (dashed line on Chart 2). Now suppose that ε_t is a white noise process with variance equal to 2π so that the spectrum of ε_t is equal to one at each frequency. We consider this example because in that case the spectrum of the cyclical component of y_t identified by the HP filter corresponds to the squared gain calculated by Cogley and Nason since the spectrum of $f_\varepsilon(\omega) = 1$. Indeed, by equation (2), when $f_\varepsilon(\omega) = 1$, the cyclical component identified by the HP filter is equal to $|HP(\omega)|^2|1-\exp(-i\omega)|^{-2}$ which corresponds exactly to the squared gain calculated by Cogley and Nason.

Chart 2 presents the pseudo-spectrum of y_t and the spectrum of the cyclical component identified by the HP filter for business cycle frequencies. We can see that the conclusion concerning the effect of the HP filter is quite different depending on whether we are interested in retrieving the component corresponding to business cycle frequencies for the level of the series y_t or for the series in difference $(1-L)y_t$.[5] In the later case, as noted by Cogley and Nason and Harvey and Jaeger, the HP filter greatly amplifies business cycle frequencies and creates spurious cycles. In the case of the series in level, the spectrum of the cyclical component identified by the HP filter has a peak corresponding to a period of 30 quarters which is absent from the spectrum of the original series while the filter reduces the importance of business cycle frequencies.

5. Some may have problems with the fact that we are interested in extracting business cycles frequencies from the level of integrated series. Note that we could also consider an AR(1) process with a coefficient of 0.95 and obtain the same result.

The conclusions of Cogley and Nason and Harvey and Jaeger concerning the cyclicality induced by the filter are valid to the extent that we are interested in the cyclical component resulting from the business cycles frequencies of the series in difference, but does not hold if we want to extract business cycle frequencies of time series in levels. Indeed, if one is to judge the performance of the HP filter by how well it does in extracting a specified range of periodicities, which is one of six objectives Baxter and King (1995) try to meet with their band-pass filter, the spectrum of the extracted component should be compared with that of the series in levels. Also, the conclusion of Cogley and Nason and Harvey and Jaeger may not hold if one is interested in the cyclical component of processes other than random walks. We consider these points in Sections 3 and 4.

CHART 2: spectrum of y_t and of that series HP filtered
(at frequencies between 6 and 32 quarters)



2.2 THE BK FILTER

While an ideal high-pass filter removes low frequencies from the data, an ideal band-pass filter removes both low and high frequencies. Baxter and King (1995) propose a finite moving average approximation of an ideal band-pass filter based on the Burns and Mitchell (1946) definition of a business cycle, i.e., their filter is designed to pass through components of time series with fluctuations between six and thirty-two quarters, while removing components at higher and lower frequencies.

When applied to quarterly data, the band-pass filter proposed by Baxter and King takes the form of a 24 quarter moving average

$$y_t^f = \sum_{h=-12}^{12} a_h y_{t-h} = a(L)\, y_t$$

The weights a_h can be derived from the inverse Fourier transform of the frequency response function -- see Priestley (1981) p. 274. When using the BK filter, twelve quarters are sacrificed at the beginning and the end of the time series, seriously limiting its usefulness for analysing contemporary data.

To study some time and frequency domain properties of the BK filter, assume the following data generating process for y_t:

$$y_t = (1-L)^{-r}\varepsilon_t \tag{3}$$

where r determines the order of integration of y_t and ε_t is a zero mean stationary process. Baxter and King show that their filter can be factorized as

$$a(L) = (1-L)^2 a^*(L)$$

thus the filter contains two unit differencing operators, enabling it to render stationary time series that contain up to two unit roots.

CHART 3a: autocorrelations corresponding to the BK filter

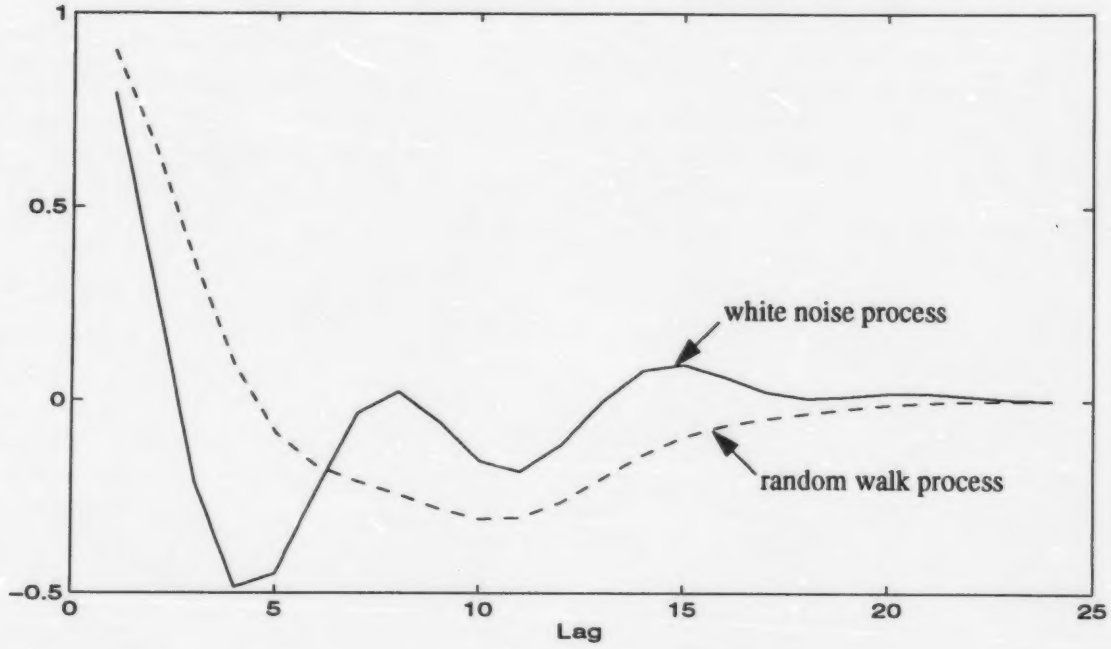


Chart 3b: autocorrelations corresponding to the HP filter

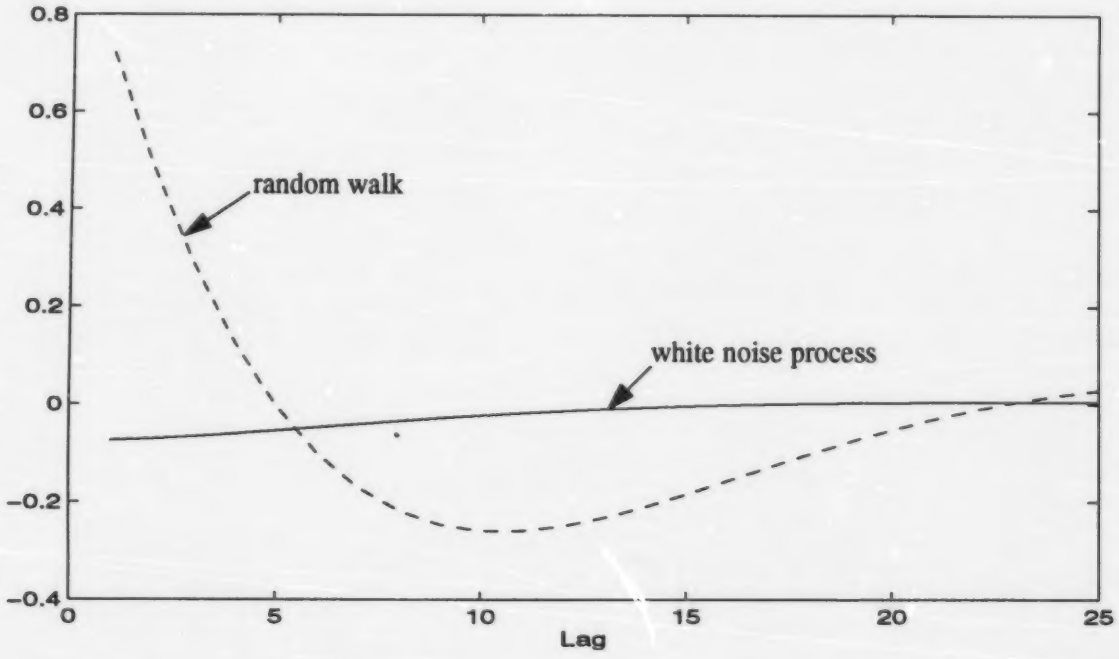


Chart 3a shows the autocorrelation functions for the BK-filtered version of a white noise process and a random walk process. In both cases, the cyclical component identified by the BK filter possesses strong positive autocorrelations at shorter horizons. The result for the random walk is similar to what Cogley and Nason (1994) find for the HP filter (this is shown in Chart 3b). However, in contrast with the HP filter, the cyclical component identified by the BK filter displays strong dynamics for a white noise process. This result precludes the use of this band-pass filter to evaluate the internal dynamic propagation mechanism of business cycle models on the basis of autocorrelation functions resulting from this filter.

The spectrum of the cyclical component obtained via applying the BK filter is

$$f_{y^c}(\omega) = |BK(\omega)|^2 f_y(\omega)$$

where $|BK(\omega)|^2$ is the squared gain of the BK filter and $f_y(\omega)$ is the spectrum of y_t. The squared gain $|BK(\omega)|^2$ is equal to $|a(\omega)|^2$ where $a(\omega)$ denotes the Fourier transform of $a(L)$ at frequency ω. The pseudo-spectrum of y_t is equal to

$$f_y(\omega) = |1 - \exp(-i\omega)|^{-2r} f_\varepsilon(\omega) = 2^{-2r}(\sin^2(\omega/2))^{-r} f_\varepsilon(\omega)$$

for $\omega \neq 0$ (see Priestley p. 597), where $f_\varepsilon(\omega)$ is the spectrum of the process ε_t which is well defined since ε_t is stationary.

Chart 4a presents the squared gain of the BK filter and compares it to the squared gain of the ideal filter. The BK filter is designed to extract low and high frequencies from the data. This is basically what we obtain. The filter passes through most components with fluctuations of between six and thirty-two quarters (respectively $\pi/3$ and $\pi/16$), while removing components at higher and lower frequencies. However, the BK filter does not exactly correspond to the ideal band-pass filter (also shown on the graph) because it is a finite approximation of an infinite moving average filter. In particular, at lower and higher frequencies we observe a compression effect so that the squared gain is less than one.

As in Section 2.1, we now assume that r=1 and that ε_t is white noise with variance equal to 2π in equation (3 The spectrum of ε_t is then equal to one at business cycle frequencies and the cyclical component obtained with the BK filter corresponds exactly to the squared gain of the BK filter as calculated by

Cogley and Nason and Harvey and Jaeger for the HP filter:

$$|BK(\omega)|^2|1 - \exp(-i\omega)|^{-2} = |BK(\omega)|^2 2^{-2}(sin^2(\omega/2))^{-1}$$

Chart 4b presents the pseudo-spectrum of y_t and the spectrum of the cyclical component identified by the BK filter at business cycle frequencies. The conclusion is again quite different depending on whether we are interested in retrieving the component corresponding to business cycle frequencies for the level of the series y_t or for the series in difference $(1 - L)y_t$. In the later case, as noted by Cogley and Nason and Harvey and Jaeger for the HP filter, the BK filter greatly amplifies business cycle frequencies and creates spurious cycles. For example, it amplifies by a factor of ten the variance of cycles with a periodicity of around 20 quarters ($\pi/10$). Also, as in the case of the HP filter, business cycle frequencies of the BK filtered series are less important than those of the original series in level and the cyclical component identified by the BK filter has a peak corresponding to a period of 20 quarters (compared with 30 quarters in the case of the HP filter) which is absent from the spectrum of the level of the series y_t.

CHART 4a: squared gain of the BK filter

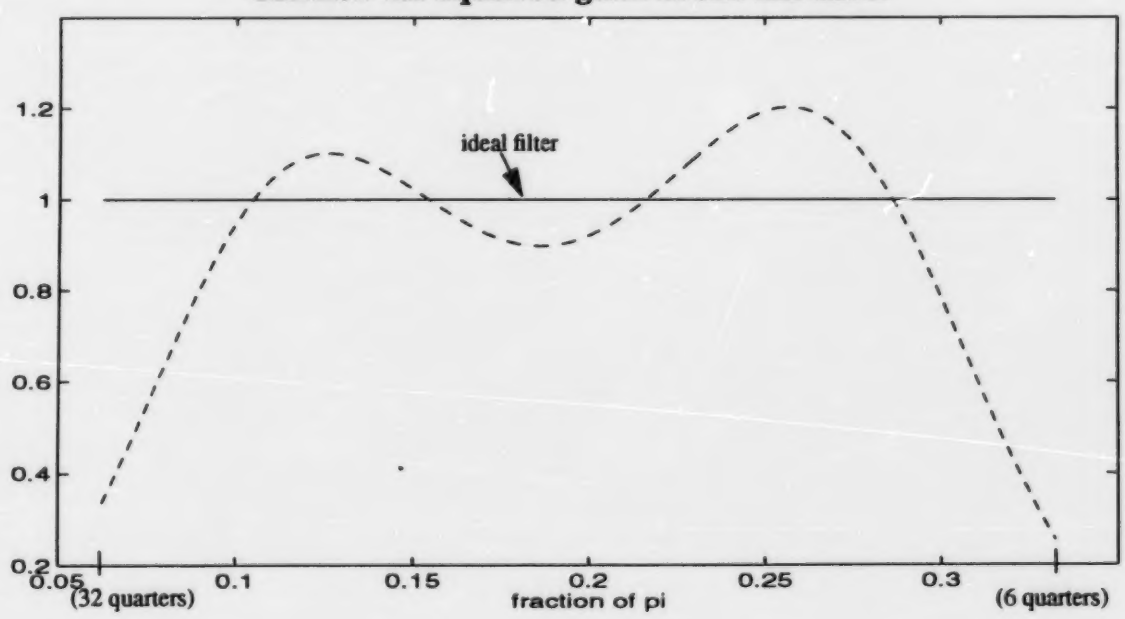
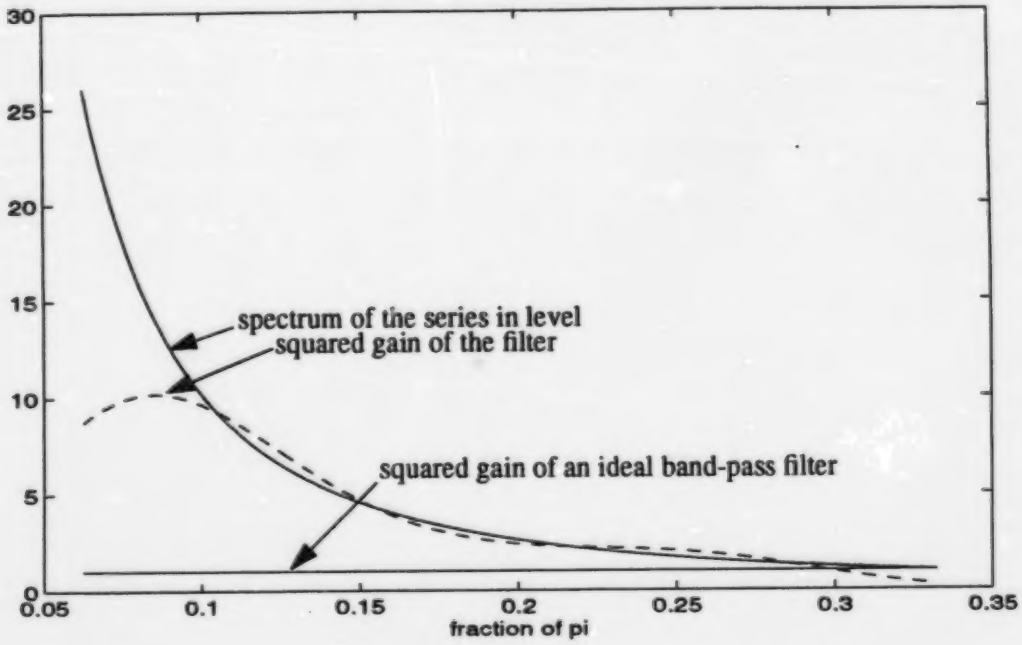


CHART 4b: squared gain of the BK filter
(the case of a random walk process)



3. ABILITY OF THE FILTERS TO EXTRACT CYCLICAL PERIODICITIES

In this section, we examine how well the BK and HP filters capture the cyclical component of macroeconomic time series. Baxter and King's (1995) first objective is to adequately extract a specified range of periodicities without altering the properties of this extracted component. We use the same criteria to assess the performance of the HP and BK filters. We show that when the peak of the spectral density function of these series lies within business cycle frequencies, these filters provide a good approximation of the corresponding cyclical component. If the peak is located at zero frequency, so that the bulk of the variance is located in low frequencies, those filters cannot identify the cyclical component adequately.

To show this, we consider the following data generating process (DGP),

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \qquad (4)$$

where $\phi_1 + \phi_2 < 1$. A second order autoregressive process is useful for our purpose

because its spectrum may have a peak at business cycle frequencies or at zero frequency. The spectrum of this process is equal to

$$f_y(\omega) = \frac{\sigma_\varepsilon^2}{1+\phi_1^2+\phi_2^2-2\phi_1(1-\phi_2)\cos\omega-2\phi_2\cos 2\omega}$$

and the location of its peak is given by

$$-\sigma_\varepsilon^{-2} f_y(\omega)^2\,((2\sin\omega)\,[\phi_1(1-\phi_2)+4\phi_2\cos\omega])$$

Thus, $f_y(\omega)$ has a peak at frequencies other than zero for

$$\phi_2 < 0 \text{ and } \left|\frac{-\phi_1(1-\phi_2)}{4\phi_2}\right| < 1 \qquad (5)$$

then $f_y(\omega)$ has a peak at $\omega = cos^{-1}(-\phi_1(1-\phi_2)/4\phi_2)$ -- see Priestley (1981). For other parameter values, the spectrum has a trough at non-zero frequencies if $\phi_2 > 0$ and $|\phi_1(1-\phi_2)/4\phi_2| < 1$.

Charts 5 and 6 show the spectrum of autoregressive processes and the spectrum of the cyclical component identified with the HP and BK filters. When the peak is located at zero frequency, i.e. most of the power of the series is located at low frequencies, the spectrum of the cyclical component resulting from the application of both filters is very different from that of the original series, especially at lower frequencies (Chart 5). In particular, the HP and BK filters induce a peak at business cycle frequencies even though it is absent from the original series. On the other hand, when the peak is located at business cycle frequencies, the spectrum of the cyclical component identified by HP and, to a lesser extent, BK filtering matches fairly well the true spectrum at these frequencies (Chart 6). This result is robust for different sets of parameters ϕ_1 and ϕ_2. Note that the absence of a peak at business cycle frequencies does not imply that macroeconomic series do not feature business cycles -- see Sargent (1987) for a discussion of this. In fact, while most macroeconomic series feature the Granger typical shape, the growth rate of these series is often characterized by a peak at the business cycle frequencies. King and Watson (1996) call this "the typical spectral shape of growth rates."

CHART 5: series having Granger's typical shape
(AR(2) coefficients: 1.26 -0.31)

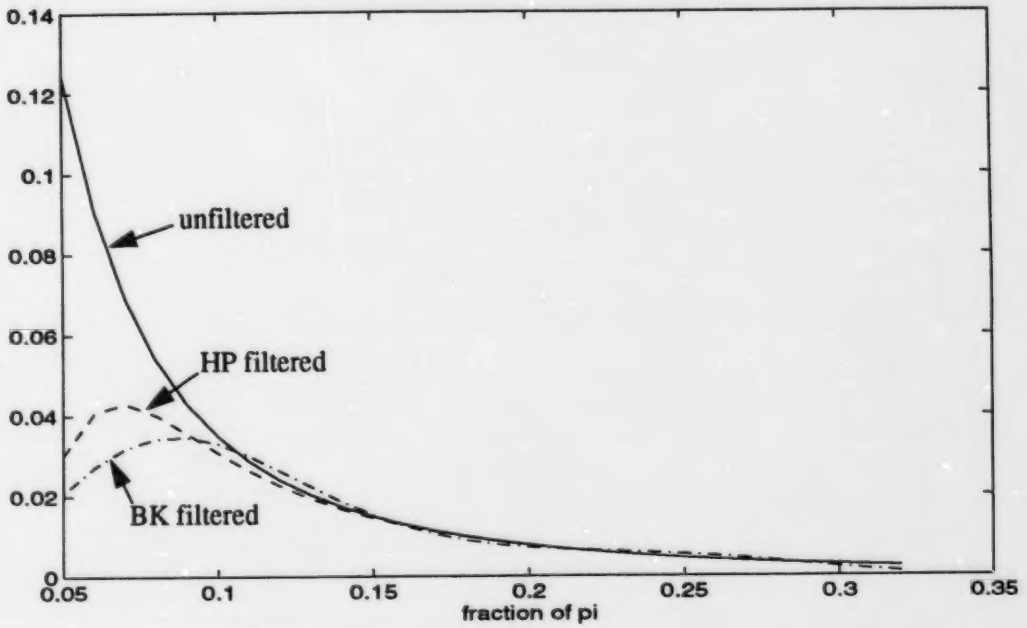
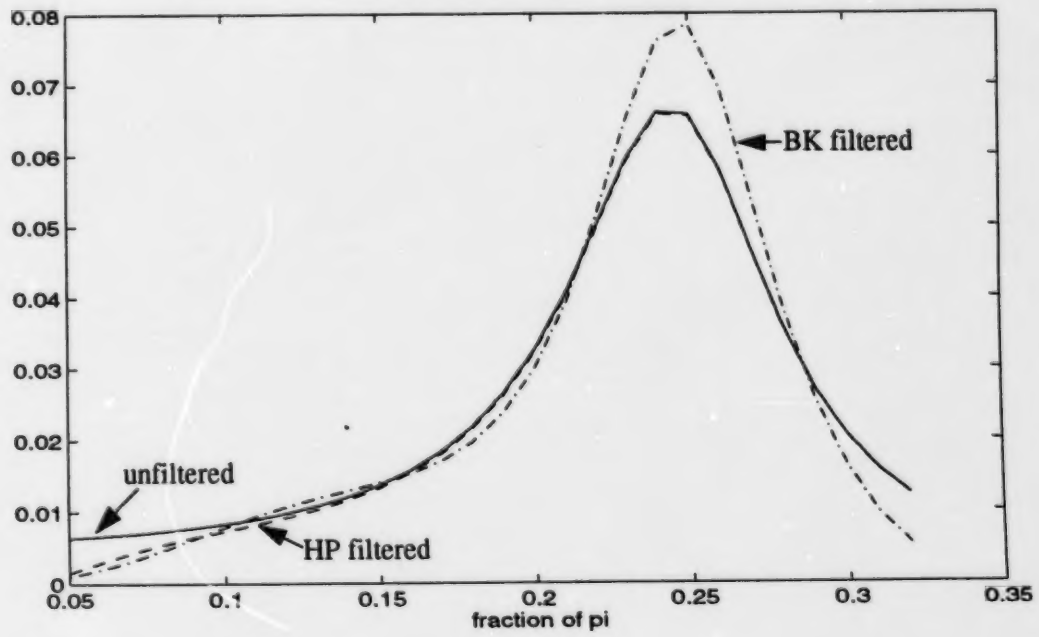


CHART 6: series with a peak at business cycle frequencies
(AR(2) coefficients: 1.26 -0.78)



To examine this question in more details, we perform the following exercise. First, we set a data generating process by a choice of $\theta = (\phi_1, \phi_2)$ for the second order autoregressive process of equation (4). Second, we extract the corresponding cyclical component with the HP or the BK filters. Third, we search among second order processes for the parameters ϕ_1 and ϕ_2 minimizing the distance, at business cycle frequencies, between the spectrum of this process and that of the HP or BK filtered true second order autoregressive processes. The problem is the following

$$\tilde{\theta} = \arg min \int_{\omega 1}^{\omega 2} (S_{y'}(\omega;\theta_0) - S_{\tilde{y}}(\omega;\theta))^2 d\omega$$

where the vector θ contains parameters ϕ_1 and ϕ_2, $\omega 1 = \pi/16$, $\omega 2 = \pi/3$, $S_{y'}(\omega;\theta_0)$ is the spectrum of the filtered DGP (where θ_0 is the vector of true values for the parameters ϕ_1 and ϕ_2), and $S_{\tilde{y}}(\omega;\theta)$ is the spectrum of the evaluated autoregressive process. Thus, in the case where the HP and BK filters extract adequately the range of periodicities corresponding to fluctuations of between six and thirty-two quarters (respectively $\omega = \pi/3$ and $\omega = \pi/16$), $\tilde{\theta}$ will be equal to the true vector θ_0. Otherwise, the filter will extract a cyclical component corresponding to a second order autoregressive process differing from the true one.

Table 1 presents our results in the case of a DGP for which the autoregressive parameter of order 1 are set at 1.20 while the parameter of order 2 is allowed to vary. Using the restrictions implied by (5), the peak of the spectrum lies within business cycle frequencies when $\phi_2 < -0.43$. We report results only for the HP filter but these are almost identical to those with the BK filter.[6] Results from this exercise corroborate those obtained from visual inspection. The second order autoregressive process which minimizes the distance between its spectrum at business cycle frequencies and that of the business cycle component identified by the HP and BK filters for the true process is very different from the true second order autoregressive process when the peak of the DGP is located at zero frequency. When the peak is located at business cycle frequencies, the resulting second order autoregressive process is close to the true second order autoregressive process.

6. These results are robust to the use of alternative values for θ_0, so that the restrictions are respected.

TABLE 1: Fitted values for the HP filter

DGP (θ_0)		HP ($\bar{\theta}$)	
ϕ_1	ϕ_2	ϕ_1	ϕ_2
1.20	-0.25	-0.09	0.72
1.20	-0.30	0.12	0.40
1.20	-0.35	0.48	-0.15
1.20	-0.40	0.87	-0.20
1.20	-0.45	1.09	-0.41
1.20	-0.50	1.16	-0.50
1.20	-0.55	1.19	-0.56
1.20	-0.60	1.20	-0.61
1.20	-0.65	1.20	-0.66
1.20	-0.70	1.20	-0.70
1.20	-0.75	1.20	-0.75
1.20	-0.80	1.20	-0.80

As noted previously, the spectrum of the level of macroeconomic time series typically looks like that of the unfiltered series shown on Chart 5. The spectrum's peak is located at zero frequency and the bulk of its variance is located in the low frequencies. Charts 7, 8, 9 and 10 display the estimated spectrum of U.S. real GDP, real consumption, consumer prices inflation, and the unemployment the rate as well as the spectrum of the filtered counterparts to these series.[7] It is clear that the filters perform badly in terms of capturing business cycle frequencies in these cases.

7. We use a parametric estimator of the spectrum. An autoregressive process was fitted and the order of that process was determined on the basis of the Akaike criteria.

The intuition behind this result is simple. Charts 1 and 4a (Section 2) show that the gains of the HP and BK filters at low business cycle frequencies are significantly smaller than that of the ideal filter. Indeed, the squared gain of the BK filter is 0.34 at frequencies corresponding to 32 quarters cycles, while that of the HP filter is 0.49. In the case of the HP filter, the squared gain does not reach 0.95 before frequency $\pi/8$ (cycles of 16 quarters). The problem is that a large fraction of the power of typical macroeconomic time series at business cycle frequencies is concentrated in the band where the squared gains of HP and BK filters differ from that of an ideal filter. Also, the shape of the squared gain of those filters when applied to typical macroeconomic time series induces a peak in the spectrum of the cyclical component which is absent from the original series. The result of applying the HP and BK filters is the extraction of a cyclical component which does not capture an important fraction of the variance contained in business cycle frequencies of the original series and which induces spurious dynamic properties.

One could argue that macroeconomic time series are really made of a permanent component and a cyclical component, so that the peak of the spectrum of the series would be at zero frequency while the peak of the spectrum of the cyclical component would be located at business cycle frequencies. For example, the permanent component could be driven by a random walk technological process with drift while transitory monetary or fiscal policy shocks, among others, would generate the cyclical component with a peak in its spectrum at business cycle frequencies. If this were true, then the HP and BK filters might be able to adequately capture the cyclical component. We examine this issue in the next section.

CHART 7: spectrum of the logarithm of US real GDP

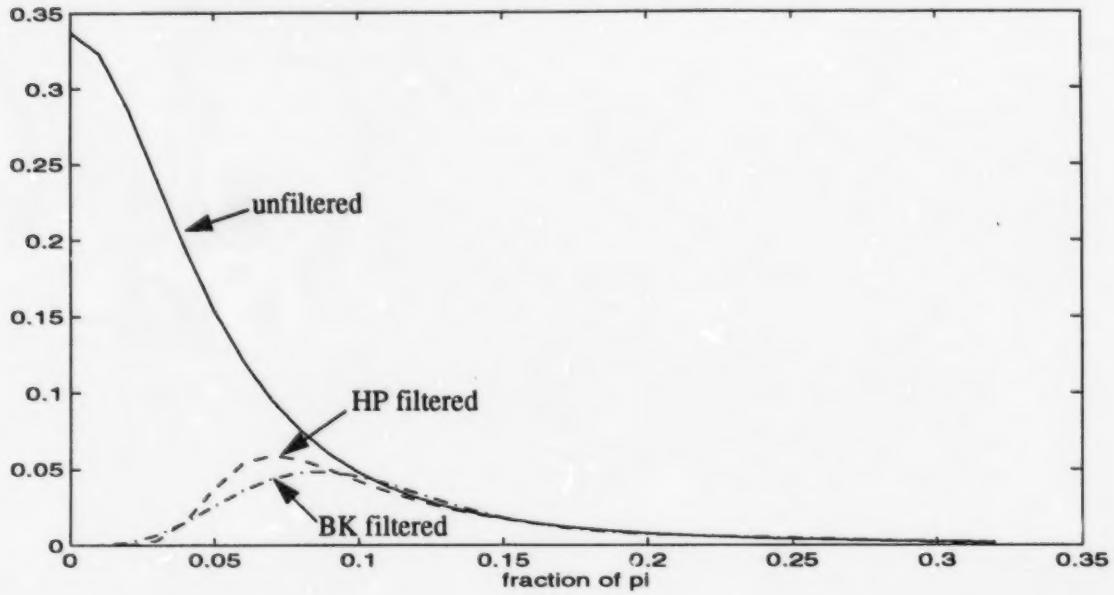


CHART 8: spectrum of the logarithm of US real consumption

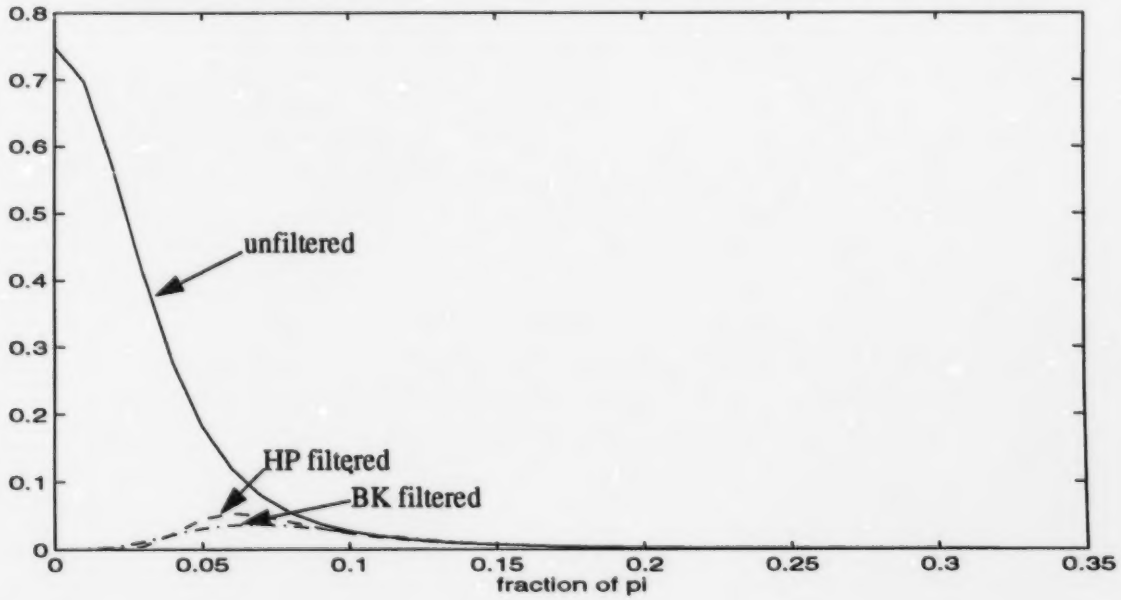


CHART 9: spectrum of US consumer prices inflation

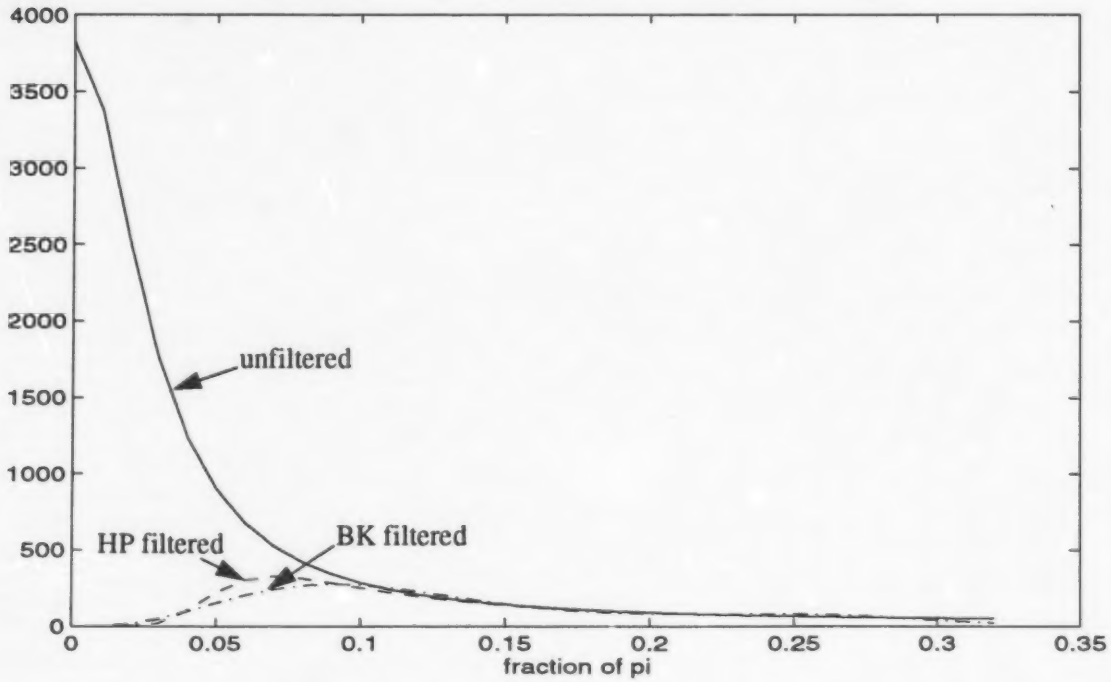
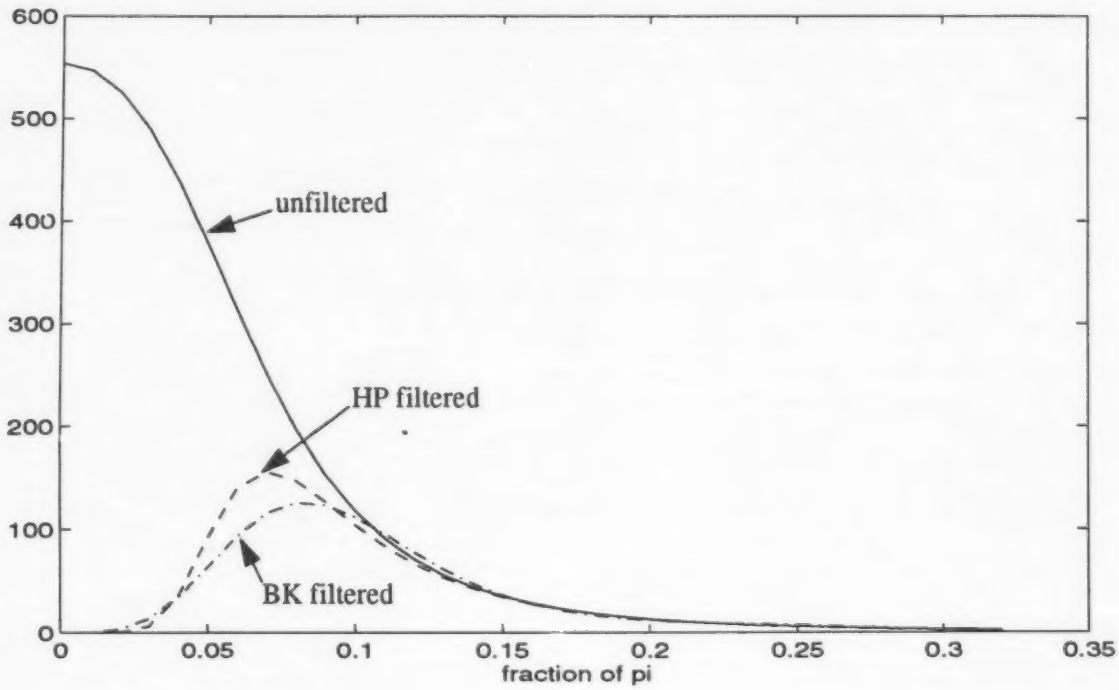


CHART 10: spectrum of US unemployment rate



4. FILTERS' ABILITY TO IDENTIFY THE CYCLICAL COMPONENT

In this section, simulation results show that the HP and BK filters perform poorly in terms of identifying the cyclical component of time series dominated by zero frequency, which is the case for most macroeconomic time series in level.

Consider the following data generating process:

$$y_t = \mu_t + c_t \tag{6}$$

where

$$\mu_t = \mu_{t-1} + \varepsilon_t$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \eta_t$$

and

$$\varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right), \eta_t \sim NID\left(0, \sigma_\eta^2\right)$$

Equation (6) defines y_t as the sum of a permanent component, μ_t, which in this case corresponds to a random walk, and a cyclical component, c_t.[8] The dynamics of the cyclical component are specified as a second order autoregressive process so that the peak of the spectrum could be at zero frequency or at business cycle frequencies. We assume that ε_t and u_t are uncorrelated, which should favour the HP filter.

Data are generated from equation (6) with ϕ_1 set at 1.2 and different values for ϕ_2 to control the location of the peak in the spectrum of the cyclical component. We also vary the standard error ratio for the disturbances $\sigma_\varepsilon/\sigma_\eta$ to change the relative importance of each component. We follow the standard practice of giving the value 1,600 to λ, the HP filter smoothness parameter. We also follow Baxter and King's suggestion of dropping 12 observations at the beginning and at the end of the sample. The resulting series contains 150 observations, a standard size for quarterly macroeconomic data. The number of replications is 500.

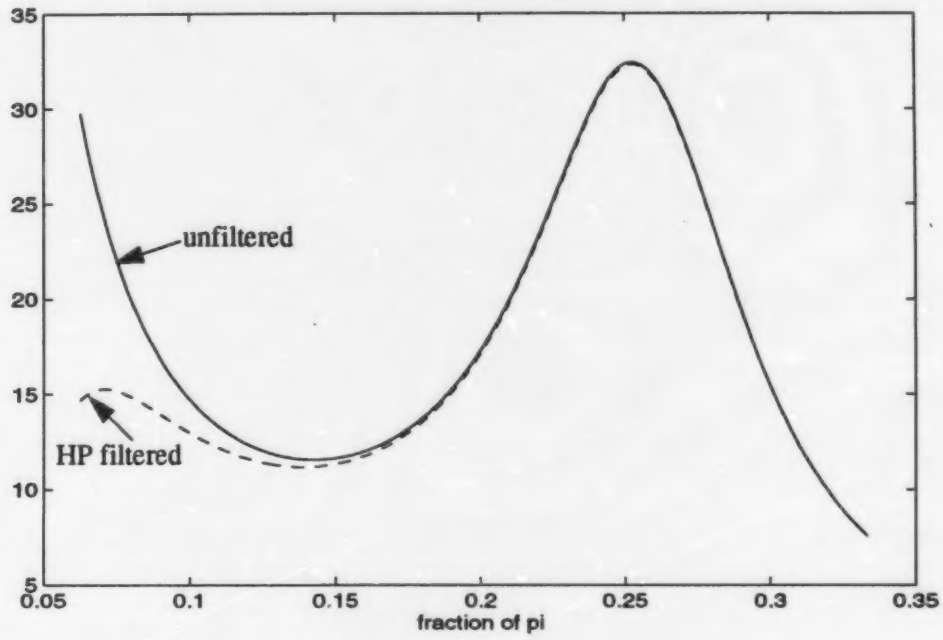
8. This is compatible with the definition of cycle used by Watson (1986).

The performance of the HP and BK filters is assessed by comparing the autocorrelation function of the cyclical component of the true process to the one obtained from the filtered data. We also calculate the correlation between the true cyclical component and the filtered cyclical component and report their relative standard deviations ($\hat{\sigma}_c/\sigma_c$). Table 2 presents the results for the HP filter and Table 3 those for the BK filter.

Table 2 shows that the HP filter performs particularly poorly when there is an important permanent component. Indeed, for high $\sigma_\varepsilon/\sigma_\eta$ ratios, in most cases, the correlation between the true and the filtered components is not significantly different from zero. The estimated autocorrelation function is invariant to the change in the cyclical component in these cases (values of the true autocorrelation functions are between brackets in the tables). When the ratio $\sigma_\varepsilon/\sigma_\eta$ is equal to 0.5 or 1 and the peak of the cyclical component is located at zero frequency ($\phi_2 < -0.43$), the dynamic properties of the true and the filtered cyclical components are significantly different as indicated by the estimated parameter values. In general, the HP filter adequately characterizes the series dynamics when the peak of the spectrum is at business cycle frequencies and the ratio $\sigma_\varepsilon/\sigma_\eta$ is very small. However, even when the ratio of standard deviations is equal to 0.01, i.e. the permanent component is almost absent, the filter performs poorly when the peak is at zero frequency. Indeed, for $\phi_2 = -0.25$, the dynamic properties of the filtered component differ significantly from those of the true cyclical component, the correlation is only equal to 0.66, and the standard deviation of the filtered cyclical component is half that of the true one.

It is interesting to note that the HP filter does relatively well when the ratio $\sigma_\varepsilon/\sigma_\eta$ is equal to 1, 0.5, or 0.01 and the spectrum of the original series has a peak at zero frequency and at the business cycle frequencies, i.e. the later contains a significant part of the variance of the series. To get a better feel for this, consider Chart 11 which shows the spectrum for the case where $\sigma_\varepsilon/\sigma_\eta = 1$ and $\phi_2 = -0.75$. Consequently, the conditions required to adequately identify the cyclical component with the HP filter can be expressed in the following way: the spectrum of the original series must have a peak located at business cycle frequencies and the later must account for an important part of the variance of the series. If the variance of the series is dominated by low frequencies, which is the case for most macroeconomic series in levels, the HP filter performs poorly in terms of extracting the cyclical component.

CHART 11



The results for the BK filter are similar to those for the HP filter, although the dynamic properties of the filtered cyclical component seem to be invariant (or almost invariant) to the true process. For example, when $\sigma_\varepsilon/\sigma_u = 0.01$, $\phi_1 = 0$, and $\phi_2 = 0$, which corresponds to the case where the cyclical component is white noise and dominates the permanent component, the filtered cyclical component is a highly autocorrelated process. Thus, the BK filter would appear to be of limited value to identify the cyclical dynamics of a macroeconomic time series with any degree of confidence. Moreover, this result precludes the use of the BK filter for assessing the internal dynamic properties of a business cycle model since the filter produces a series with dynamic properties that are almost invariant to the true process.

TABLE 2: HP filter

DGP			Estimated values				
$\sigma_\varepsilon/\sigma_\eta$	ϕ_1	ϕ_2	Autocorrelations			correlation	$\hat{\sigma}_c/\sigma_c$
			1	2	3		
10	0	0	.71[0] (.59,.80)	.46[0] (.30,.60)	.26[0] (.08,.43)	.08 (-.07,.21)	12.96 (10.57,15.90)
10	1.2	-.25	.71[.96] (.61,.80)	.47[.90] (.31,.61)	.27[.84] (.08,.44)	.08 (-.11,.28)	4.19 (2.77,6.01)
10	1.2	-.40	.71[.86] (.60,.80)	.46[.63] (.30,.60)	.26[.41] (.08,.44)	.13 (-.12,.36)	6.34 (4.82,8.07)
10	1.2	-.55	.71[.77] (.60,.80)	.46[.38] (.29,.60)	.26[.03] (.06,.43)	.14 (-.08,.33)	6.93 (5.36,8.70)
10	1.2	-.75	.71[.69] (.60,.78)	.46[.27] (.30,.59)	.25[-.19] (.07,.41)	.15 (-.01,.31)	6.37 (4.79,7.95)
5	0	0	.69[0] (.58,.78)	.45[0] (.30,.58)	.26[0] (.09,.41)	.15 (.02,.27)	6.50 (5.28,7.85)
5	1.2	-.25	.71[.96] (.61,.80)	.46[.90] (.32,.61)	.26[.84] (.08,.43)	.16 (-.01,.36)	2.11 (1.43,3.04)
5	1.2	-.40	.72[.86] (.61,.80)	.46[.63] (.31,.60)	.25[.41] (.08,.42)	.23 (-.01,.45)	3.26 (2.47,4.15)
5	1.2	-.55	.71[.77] (.61,.80)	.46[.38] (.30,.59)	.24[.03] (.06,.41)	.24 (.01,.44)	3.60 (2.83,4.52)
5	1.2	-.75	.70[.69] (.61,.79)	.43[.27] (.26,.57)	.20[-.19] (.00,.38)	.29 (.11,.44)	3.30 (2.53,4.17)
1	0	0	.43[0] (.27,.57)	.28[0] (.11,.42)	.20[0] (-.02,.31)	.59 (.49,.70)	1.61 (1.41,1.85)
1	1.2	-.25	.76[.96] (.67,.83)	.51[.90] (.37,.62)	.29[.84] (.11,.44)	.51 (.33,.68)	.66 (.44,.91)
1	1.2	-.40	.75[.86] (.67,.81)	.44[.63] (.28,.55)	.16[.41] (-.03,.33)	.71 (.56,.82)	1.02 (.83,1.22)
1	1.2	-.55	.72[.77] (.66,.78)	.34[.38] (.21,.47)	.01[.03] (-.17,.19)	.76 (.56,.82)	1.15 (.83,1.22)
1	1.2	-.75	.68[.69] (.63,.72)	.15[.27] (.04,.27)	-.27[-.19] (-.44,.10)	.83 (.75,.89)	1.16 (1.04,1.29)
.5	0	0	.16[0] (.01,.32)	.10[0] (-.04,0.24)	.04[0] (-.10,.18)	.82 (.75,.88)	1.16 (1.07,1.27)
.5	1.2	-.25	.79[.96] (.71,.85)	.53[.90] (.38,.65)	.30[.84] (.11,.46)	.61 (.41,.79)	.55 (.37,.76)
.5	1.2	-.40	.77[.86] (.69,.81)	.43[.63] (.29,.54)	.13[.41] (-.05,.29)	.84 (.73,.92)	.87 (.74,.99)
.5	1.2	-.55	.72[.77] (.67,.78)	.28[.38] (.17,.39)	-.10[.03] (-.25,.06)	.89 (.83,.94)	.98 (.89,1.07)
.5	1.2	-.75	.67[.69] (.63,.71)	.07[.27] (-.03,.18)	-.42[-.19] (-.57,-.27)	.94 (.90,.96)	1.02 (.97,1.08)

TABLE 2: HP filter

.01	0	0	-.08[0] (-.21,.06)	-.06[0] (-.21,.06)	-.06[0] (-.19,.06)	.98 (.96,.99)	.97 (.94,.99)
.01	1.2	-.25	.80[.96] (.72,.86)	.54[.90] (.38,.67)	.30[.84] (.11,.48)	.66 (.45,.83)	.51 (.34,.69)
.01	1.2	-.40	.78[.86] (.72,.83)	.43[.63] (.30,.55)	12[.41] (-.05,.28)	.90 (.82,.96)	.81 (.71,.90)
.01	1.2	-.55	.73[.77] (.67,.77)	.26[.38] (.15,.37)	-.14[.03] (-.30,.01)	.96 (.91,.99)	.92 (.86,.96)
.01	1.2	-.75	.67[.69] (.62,.71)	.02[.27] (-.08,.13)	-.50[-.19] (-.61,-.35)	.99 (.97,1.0)	.97 (.95,.99)

TABLE 3: BK filter

DGP			Estimated values				
$\sigma_\varepsilon/\sigma_\eta$	ϕ_1	ϕ_2	Autocorrelations			corr	$\hat{\sigma}_c/\sigma_c$
			1	2	3		
10	0	0	.90[0] (.87,.93)	.65[0] (.52,.75)	.33[0] (.13,.51)	.03 (-.11,.16)	11.55 (9.05,14.38)
10	1.2	-.25	.90[.96] (.87,.93)	.65[.90] (.55,.74)	.34[.84] (.17,.49)	.08 (-.13,.32)	3.71 (2.34,5.45)
10	1.2	-.40	.90[.86] (.87,.93)	.64[.63] (.54,.73)	.33[.41] (.16,.48)	.11 (-.16,.36)	5.67 (4.19,7.18)
10	1.2	-.55	.90[.77] (.87,.93)	.64[.38] (.53,.73)	.33[.03] (.14,.48)	.12 (-.12,.33)	6.23 (4.71,7.93)
10	1.2	-.75	.90[.69] (.86,.92)	.63[.27] (.52,.73)	.31[-.19] (.13,.48)	.16 (-.04,.36)	5.69 (4,37,7.16)
5	0	0	.90[0] (.87,.90)	.64[0] (.53,.73)	.33[0] (.14,.49)	.05 (-.09,.20)	5.80 (4.54,7.16)
5	1.2	-.25	.90[.96] (.87,.93)	.65[.90] (.54,.73)	.34[.84] (.16,.49)	.17 (-.05,.38)	1.94 (1.25,2.74)
5	1.2	-.40	.90[.86] (.87,.93)	.64[.63] (.53,.74)	.32[.41] (.14,.49)	.23 (-.03,.47)	2.93 (2.15,3.76)
5	1.2	-.55	.89[.77] (.87,.92)	.62[.38] (.52,.72)	.30[.03] (.12,.46)	.26 (.03,.46)	3.19 (2.45,3.98)
5	1.2	-.75	.88[.69] (.85,.92)	.60[.27] (.47,.70)	.26[-.19] (.06,.44)	.28 (.09,.45)	2.97 (2.24,3.77)
1	0	0	.89[0] (.85,.92)	.61[0] (.48,.71)	.27[0] (.06,.45)	.19 (.05,.32)	1.21 (.96,1.43)
1	1.2	-.25	.90[.96] (.87,.93)	.65[.90] (.53,.74)	.34[.84] (.15,.50)	.53 (.36,.71)	.60 (.39,.84)
1	1.2	-.40	.88[.86] (.85,.91)	.58[.63] (.47,.68)	.22[.41] (.03,.39)	.70 (.55,.81)	.95 (.78,1.12)
1	1.2	-.55	.85[.77] (.81,.89)	.48[.38] (.36,.60)	.05[.03] (-.15,.24)	.73 (.61,.83)	1.06 (.89,1.23)
1	1.2	-.75	.79[.69] (.75,.83)	.27[.27] (.14,.40)	-.26[-.19] (-.45,-.06)	.79 (.69,.87)	1.08 (.96,1.20)

TABLE 3: BK filter

DGP			Estimated values				
$\sigma_\varepsilon/\sigma_\eta$	ϕ_1	ϕ_2	Autocorrelations			corr	$\hat{\sigma}_c/\sigma_c$
			1	2	3		
.5	0	0	.86[0] (.81,.89)	.50[0] (.35,.63)	.10[0] (-.12,.30)	.36 (.25,.47)	.77 (.63,.91)
.5	1.2	-.25	.90[.96] (.87,.93)	.65[.90] (.55,.74)	.34[.84] (.16,.50)	.63 (.45,.78)	.51 (.34,.71)
.5	1.2	-.40	.88[.86] (.84,.91)	.56[.63] (.43,.66)	.17[.41] (-.02,.34)	.81 (.71,.88)	.81 (.67,.93)
.5	1.2	-.55	.83[.77] (.80,.87)	.41[.38] (.30,.53)	-.06[.03] (-.24,.12)	.85 (.78,.91)	.91 (.81,1.01)
.5	1.2	-.75	.76[.69] (.72,.79)	.17[.27] (.06,.29)	-.42[-.19] (-.57,-.25)	.89 (83,.93)	.96 (.89,1.03)
.01	0	0	.79[0] (.75,.83)	.29[0] (.16,.42)	-.22[0] (-.40,-.03)	.55 (.48,.63)	.51 (.43,.58)
.01	1.2	-.25	.91[.96] (.88,.93)	.66[.90] (.57,.74)	.35[.84] (.20,.50)	.68 (.52,.82)	.48 (.32,.64)
.01	1.2	-.40	.87[.86] (.84,.90)	.54[.63] (.44,.64)	15[.41] (-.03,.31)	.86 (.79,.92)	.76 (.65,.86)
.01	1.2	-.55	.83[.77] (.79,.86)	.39[.38] (.27,.49)	-.11[.03] (-.29,.06)	.90 (.85,.94)	.86 (.78,.92)
.01	1.2	-.75	.74[.69] (.71,.78)	.13[.27] (.03,.23)	-.48[-.19] (-.61,-.34)	.93 (.89,.96)	.92 (.86,.97)

5. COMPARISON WITH OTHER APPROACHES

In this section, we compare the cyclical component obtained via using the HP and BK filters with those of other approaches. Watson (1986) proposes an unobserved stochastic trend decomposition into permanent and cyclical components. His model for US real GDP is the following:

$$y_t = \mu_t + c_t$$

with

$$\mu_t = \mu_{t-1} + \varepsilon_t$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + u_t$$

$$\varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right) \quad , \quad u_t \sim NID\left(0, \sigma_u^2\right)$$

It should be interesting to see whether the HP or the BK filters is able to capture the cyclical component of the above DGP. Using Kuttner's (1994) estimates ($\phi_1 = 1.44$, $\phi_2 = -0.47$, $\sigma_\varepsilon^2 = 0.0052$, and $\sigma_u^2 = 0.0069$),[9] we simulate data on the basis of this DGP, filter it, and compare the dynamic properties and the correlation of the true and the filtered components. The results are shown in Table 4. Both the HP and BK filters produce cyclical components with dynamic properties significantly different from the true one. Notably, the cyclical components identificd by both filters are much less persistent than the true one. Also, the correlation is rather small. These results are not surprising since the spectrum of the cyclical component has its peak at zero frequency and the bulk of the variance is located in the low frequencies.

TABLE 4

	Autocorrelations			Correlation
	1	2	3	
Theoretical values	0.98	0.94	0.89	--
Baxter and King's filter	0.92 (0.89-0.94)	0.70 (0.60-0.78)	0.42 (0.25-0.56)	0.47 (0.30-0.67)
Hodrick-Prescott filter	0.84 (0.78-0.89)	0.61 (0.48-0.73)	0.38 (0.20-0.54)	0.56 (0.35-0.76)

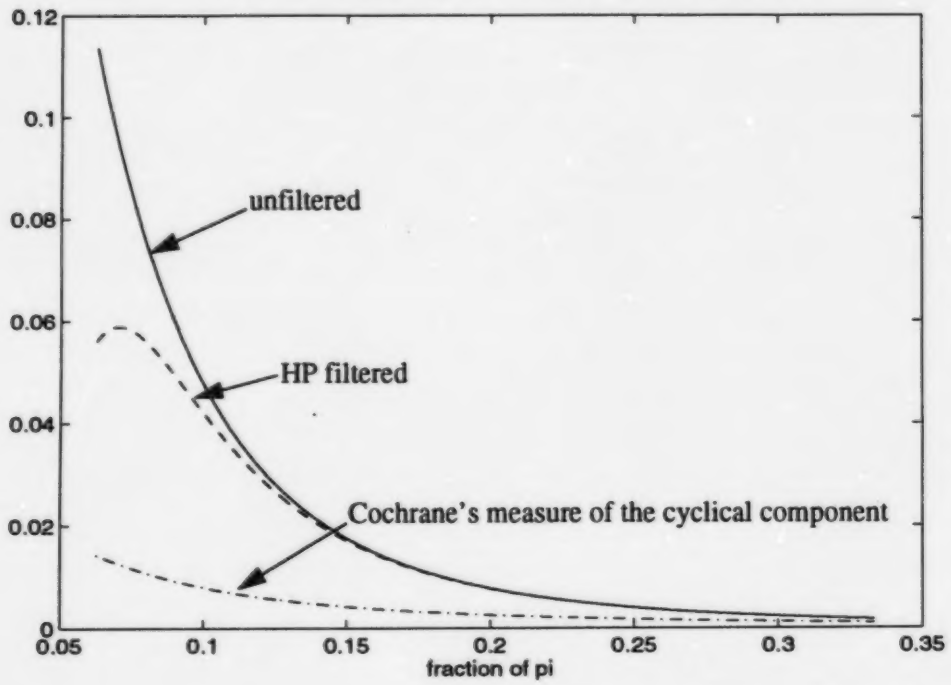
Cochrane (1994) proposes a simple detrending method for output based on the permanent income hypothesis. This implies (for a constant real interest rate) that consumption is a random walk with drift which is cointegrated with total income. Thus, any fluctuations in GDP with unchanged consumption must be transitory. Cochrane uses these assumptions to decompose US real GDP

9. We chose Kuttner's estimates because he uses a larger sample than Watson. The use of Watson's estimates would not change our conclusions.

into permanent and transitory components. Chart 12 presents the spectrum for US real GDP, for this series when it is HP filtered, and for Cochrane's cyclical component.

Using Cochrane's measure for comparison, the HP cyclical component greatly amplifies business cycle frequencies. Also, while the peak of the spectrum of the HP filtered cyclical component is located at business cycle frequencies, that of Cochrane's measure is at zero frequency. The correlation between the two cyclical components is 0.57. To the extent that Cochrane's method provides a good approximation of the cyclical component of US real GDP, the HP filtered measure appears inadequate.

CHART 12: spectrum of US real GDP



6. CONCLUSIONS

The difficulty of identifying the cyclical component of economic series has been emphasized in recently published research. For example, Quah (1992) notes that there is an infinite number of ways in which a given series can be decomposed into cyclical and permanent components. Therefore, identifying assumptions have to be used in order to construct a meaningful measure of these components.

This paper shows that the assumptions embedded in the HP filter and Baxter and King's band-pass filter limits their usefulness for identifying trend and cyclical components in macroeconomic time series. While the HP and BK filters do relatively well when applied to series with a peak in their spectrum at business cycle frequencies, most macroeconomic time series have Granger's typical shape, i.e. they have most of their power at low frequencies and their spectrum decreases monotonically at higher frequencies. Consequently, the conditions required to obtain a good approximation of the cyclical component with the HP and BK filters are rarely met in practice. Detrending methods using economic theory as in Cochrane (1994), Blanchard and Quah (1989), and King, Plosser, Stock and Watson (1991) may be more promising.

REFERENCES

Baxter, M. 1994. "Real Exchange Rates and Real Interest Differentials: Have we Missed the Business-Cycle Relationship?" *Journal of Monetary Economics 33*: 5-37.

Baxter, M. and R. G. King. 1995. "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series." *Working Paper No. 5022*. National Bureau of Economic Research.

Blanchard, O. J. and D. Quah. 1989. "The Dynamic Effect of Aggregate Demand and Supply Disturbances." *American Economic Review* 79: 655-73.

Burns, A. M. and W. C. Mitchell. 1946. *Measuring Business Cycles*.

Cecchetti S. G. and A. K. Kashyap. 1995. "International Cycles." *Working Paper No. 5310*. National Bureau of Economic Research.

Cochrane, J. H. 1994. "Permanent and Transitory Components of GNP and Stock Prices." *Quarterly Journal of Economics 61*: 241-65.

Cogley, T. and J. Nason. 1995. "Effects of the Hodrick-Prescott filter on Trend and Difference Stationary Time Series: Implications for Business Cycle Research." *Journal of Economic Dynamics and Control 19*: 253-78.

Harvey, A. C. and A. Jaeger. 1993. "Detrending, Stylized Facts and the Business Cycle." *Journal of Applied Econometrics 8*: 231-47.

Hasler, J., P. Lundvik, T. Persson, and P. Soderlind. 1994. "The Swedish Business Cycle: Stylized Facts Over 130 Years." In *Measuring and Interpreting Business Cycles*, ed. V. Berstrom and A. Vredin. Oxford: Clarendon Press, pp. 7-108.

Hodrick, R. J. and E. Prescott. 1980. "Post-war U. S. Business Cycles: An Empirical Investigation." Working Paper. Carnegie-Mellon University.

Howrey, E.P. 1968. "A Spectral Analysis of the Long-swing Hypothesis." *International Economic Review 2*: 228-60.

King, R. G., G. H. Plosser, J. H. Stock, and M. W. Watson. 1991. "Stochastic Trends and Economic Fluctuations." *American Economic Review* 81 (September): 819-40.

King, R. G. and S. Rebelo. 1993. "Low Frequency Filtering and Real Business Cycles." *Journal of Economic Dynamics and Control 17*: 207-31.

King, R. G., J. H. Stock and M. Watson. 1995. "Temporal Instability of the Unemployment-Inflation Relationship." *Economic Perspectives*. Federal Reserve Bank of Chicago. Vol. 19, pp. 753-95.

King, R. G., M. Watson. 1996. "Money, Prices, Interest Rates and the Business Cycle." Forthcoming in the *Review of Economics and Statistics*.

Kuttner, K. N. 1994. "Estimating Potential Output as a Latent Variable." *Journal of Business and Economic Statistics 12*: 361-68.

Laxton, D. and R. Tetlow. 1992. *A Simple Multivariate Filter for the Measurement of Potential Output*. Technical Report No. 59. Ottawa: Bank of Canada.

Nelson, C. R. and H. Kang. 1981. "Spurious Periodicity in Inappropriately Detrended Time Series." *Econometrica 49*: 741-51.

Nelson, C. R. and C. Plosser. 1982. "Trends and Ransom Walks in Macroeconomic Time Series." *Journal of Monetary Economics 10*: 139-67.

Osborn, D. R. 1995. "Moving Average Detrending and the Analysis of Business Cycles." Forthcoming in *Oxford Bulletin of Economics and Statistics.*

Priestly, M. 1987. *Spectral Analysis and Time Series.* Academic Press.

Quah, D. 1992. "The Relative Importance of Permanent and Transitory Components: Identification and Some Theoretical Bounds." *Econometrica 60*: 107-18.

Sargent, T. J. 1987. *Macroeconomic Theory.* Academic Press.

Singleton, K. 1988. "Econometric Issues in the Analysis of Equilibrium Business Cycle Models." *Journal of Monetary Economics 21*: 361-86.

Slutzky, E. 1937. "The Summation of Random Causes as the Source of Cyclic Processes." *Econometrica*: 5. 105-46.

Van Norden, S. 1995. "Why is it so Hard to Measure the Current Output Gap?" Mimeo. Bank of Canada.

Watson, M. W. 1986. "Univariate Detrending Methods with Stochastic Trends." *Journal of Monetary Economics 18:* 49-75.

LIST OF WORKING PAPERS

96-02 Do Mechanical Filters Provide a Good Approximation of Business Cycles? by **Alain Guay and Pierre St-Amant**

96-01 An Indicator Model of Core Inflation in Canada by **Robert Lamy and Paul Rochon**

95-12 Potential Economic Effects of Experience-Rating the Unemployment Insurance System Using a Multi-Sector General Equilibrium Model of Canada by **Louis Beauséjour, Munir A. Sheikh and Baxter Williams**

95-11 Active Labour Market Policies: An Evaluation by **Lori Marchildon**

95-10 An Index of UI Disincentives by **Tim Sargent**

95-9 The Economics of Canada Pension Plan Reforms by **Steven James, Chris Matier, Humam Sakhnini and Munir Sheikh**

95-8 The Long-Run Economic Impacts of Government Debt Reduction by **Steven James and Chris Matier**

95-7 Time Series Properties of Canadian Real Interest Rates by **Chris Matier and Werner Schüle**

95-6 Interpreting Sacrifice Ratios Across Countries and Over Time by **Brenda Lipsett & Steven James**

95-5 Effective Rates of Assistance: An Old Idea Whose Time Has Come? by **Louis Beauséjour and Munir A. Sheikh**

95-4 Aggregate Fluctuations, Monetary Policy and Nominal Wage Rigidities in a New-Classical Setting: Results for Canada by **Humam Sakhnini**

95-3 Ricardian Equivalence and the Permanent Income Hypothesis: An Empirical Investigation by **Benoit Perron**

95-2 Are New-Classical Models Becoming Relevant Policy Tools: A Survey of Recent New-Classical Models by **Humam Sakhnini**

95-1 Fluctuations économiques dans un environnement de stabilité des prix par **Claude Lavoie**

94-6 Conditional Productivity Convergence in Canada by **Frank C. Lee**

94-5 A Guide to Applied Modern Macroeconometrics by **Alain Paquet**

94-4 Interindustry Wage Differentials: Evidence, Explanations and Potential Distortions by **Mario Iacobacci**

94-3 Regional Productivity Convergence in Canada by **Frank C. Lee and Serge Coulombe**

94-2 Regional Economic Disparities in Canada by **Frank C. Lee and Serge Coulombe**

94-1 The Interaction of Inflation with a Canadian-Type Capital Tax System: a Dynamic General Equilibrium Analysis Incorporating Endogenous Leverage and Heterogeneous Households by **Steven James**

92-4 An Environmental CGE Model of Canada and the United States by **Louis Beauséjour, Gordon Lenjosek and Michael Smart**

92-3 Estimating Trend Total Factor Productivity Growth in Canada by **Carolyn Wilkins, Frank Lee and Steven James**

92-2 Un indice de prix régional de biens et services comparables au Canada et son application aux disparités régionales de revenue par **Raynald Létourneau**

92-1 A New Composite Leading Indicator of the Canadian Economy by **Robert Lamy**

90-7 Une analyse empirique des déterminants du taux d'intérêt réel mondial par **Benoit Robidoux**

90-6 Real Interest Rates in Historical Perspective by **Paul Jenkins**

90-5 The Contribution of Fiscal Policy to Real Interest Rate Movements in the 1980s by **Paul Jenkins**

90-4 The Constant Price Hypothesis in Multi-Industry Cross-Section Studies: The Impact on Cost Function Estimates by **Benoit Robidoux and John Lester**

90-3 Regional Disparities in Canadian Manufacturing: The Impact of Plant Size and Technology by **Benoit Robidoux and John Lester**

90-2 The Determination of Real Interest Rates by **Joel Fried and David Burgess**

90-1 La voie des dépenses fiscales: le point sur les questions conceptuelles et les controverses par **Neil Bruce et Michael Daly**

89-7 INTERMOD 2.0: Model Specification and Simulation Properties by **Guy Meredith**

89-6 The Calculation of Marginal Effective Tax Rates in the 1987 White Paper on Tax Reform by **Jack Jung**

89-5 Effects of Changing Age Structure on Consumption and Savings by **Paul Jenkins**

89-4 Expectations, Policy Shocks, and Output Behaviour in a Multi-Country Macroeconomic Model by **Guy Meredith**

89-3 The Goods and Services Tax: A General Equilibrium Analysis by **Bob Hamilton and Chun-Yan Kuo**

89-2 A Survey of Some Recent Developments in Macroeconomics by **Michael Devereux**

89-1 Closed Economy Analysis of the Dynamics of Deficits and Debt in a Unitary and Federal State by **Carolyn Wilkins and Mario Albert**

88-10 A Guide to the Get Model by **Richard G. Harris**

88-9 Intermod Model User's and Developer's Guide for PC Systems by **P. Bagnoli and A. Kruns**

88-8 The Trade Sector in INTERMOD by **Guy Meredith**

88-7 Intermod 1.1: A G-7 Version of the IMF's Multimod by **John Helliwell, Guy Meredith, P. Bagnoli and Yves Durand**

88-6 Resolutions of the U.S. Trade Imbalances: How Painful Will the Adjust Be? by **Janice Haritos**

88-5 Econometric Estimates of Import Price Elasticities for Canada by **Raynald Létourneau and John Lester**

88-4 Econometric Estimates of Scale Economies in Canadian Manufacturing by **Benoît Robidoux and John Lester**

88-3 Trade Barriers Between Canada and the United States by **John Lester and Tony Morehen**

88-2 New Estimates of Canadian Tariff Rates by Industry and Commodity by **John Lester and Tony Morehen**

88-1 Geographic Labour Mobility in Canada by **Louis Grignon and Kei Moray**

87-1 Économies d'échelle: Un survol des techniques d'estimation et des problèmes méthodologiques par **Benoît Robidoux**

86-1 The Structure of Production in Ten Canadian Industries by **François Delorme and John Lester**